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Fundamental Studies on Crashworthiness Design with Uncertainties in the System

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ABSTRACT

Previous research [1] using an advanced multi-domain topology optimization technique has shown a great promise for the crashworthiness design using the new technique. In this paper, we try to answer some fundamental questions regarding the crashworthiness design, which include: 1) what are the fundamental crash mechanisms of a general crash process; 2) how the uncertainties in the system will affect the crash behavior of a structure; and 3) what is the proper approach for the crashworthiness design optimization that will have needed effectiveness and efficiency. In this paper, three different kinds of uncertainties, uncertainties in the structural parameters, the modeling processes, and the loading and boundary conditions, will be considered to assess the effects of the uncertainties in the crash process. The possible crash mechanisms are then studied to provide an understanding for the design problem. A preliminary discussion on a systematic step by step design approach is provided, which employs linear static finite element analyses to improve the efficiency of the design process. This approach will be extended in the next step for more general crashworthiness design problems and with the use of the topology optimization.

INTRODUCTION

Vehicle crashworthiness design has been studied for decades. With the increase of higher standard in vehicle safety requirements from government and consumer, design for crashworthiness became a major task in the vehicle development process. Lots of efforts have been exerted into the crashworthiness design for automotive vehicles, as well as for airplanes and trains.

Experimental testing of crashworthiness is expensive and time-consuming, and it can only be used in the design stages when the design is close to being finalized. With the advances in computer simulation, vehicle crash simulation becomes more and more effective and can be used to partially substitute the testing. However, performing a design optimization with a full nonlinear finite element analysis model is still not practical due to the limits on computational resource and difficulties in complicated design tasks. Another issue relies on the fact that structural optimization often requires gradients of the objective and constraints to determine a search direction for the optimal solution. For a vehicle crash problem, the objective and constraints functions are often too noisy to find the gradients. Simplified models are usually used in the design processes, these simplified models can be constructed using simplified theoretical models, approximate analytic solutions, a coarse finite element mesh, etc., parametric optimization techniques by using these simplified models have been studied; examples include using the Response Surface Methods (RSM) and Design of Experiment (DOE) [2-4]. Space Mapping (SM) is another practical technique in which a surrogate model complements the full model. The surrogate model (coarse model) determines the search direction and the full model (fine model) determines the design point for the next iteration [5].

With the development of novel structural optimization methods, such as the topology optimization, a lot of progress has been achieved since the 1990's [6-8]. The revolutionary topology optimization method proposed by Bendsøe and Kikuchi [9] inaugurated a new era in structure optimization including vehicle crashworthiness design. Jeong applied this method to the crashworthiness optimization of an X-Frame [10]. Previous research [1] using a multi-domain topology optimization technique has shown a great promise for the crashworthiness design.

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The ultimate goal of this research is to develop an advanced topology optimization technique for efficient and effective crashworthiness design with multidisciplinary objectives and uncertainties in the system. The objective of this paper is to reveal some fundamental issues, including the effects of uncertainties in a crashworthiness design process. Uncertainty is an important facet overlooked in the previous studies of the crashworthiness design. An optimal topology design from a deterministic design process may no longer be optimal if the uncertainties in the structural system and loading conditions are considered. More important, uncertainties could result in a qualitatively different crash process, not like with the other structural performance measures, such as durability and NVH, uncertainties may only have quantitative effects on them, as this will be discussed in the later part of this paper. A systematic approach to achieve a robust topology design is a crucial matter for the crashworthiness design.

Simulating the nonlinear behavior of a crashing structure is a very important matter in the crashworthiness design, and in meanwhile it results in a very difficult design problem. To perform an effective and efficient topology design for crashworthiness, the design process has to be decomposed so that simpler design steps can be followed. In order to achieve this, fundamental crash mechanisms in a crash process need to be understood. In this paper, we study the general failure mechanisms of a structure by using the finite element analyses. The typical crash phenomena include progressive collapse, global buckling, and plastic hinge, which will be discussed in details.

FINITE ELEMENT MODEL USED IN CURRENT STUDY

Most of the crash energy absorption structures in modern vehicles are made with thin-walled section because of the high energy-absorption capability. In this paper, thin-walled square tubes are considered as a simplified energy absorption device, which may represent a frame front rail in a truck-type vehicle. The square tube has the baseline geometry of width $b = 80$ mm, length $l = 400$ mm and wall thickness $t = 1.5$ mm, which represents a typical stamped component in real vehicles. A total mass of 400kg is attached to an end of the tube as shown in Fig. 1. The initial velocity is 30 mph and the tube crash onto a rigid wall which has infinite mass. For studying the load direction effects, the wall is declined by an angle α . The end of the tube moves only in a longitudinal direction since it is restrained by the connected structures. The non-linear finite element analysis code LS-Dyna3D is employed for the numerical simulation. A single surface contact algorithm (Type 13 in LS-Dyna3D) is used to account for the contact between lobes of the tube during the crash.

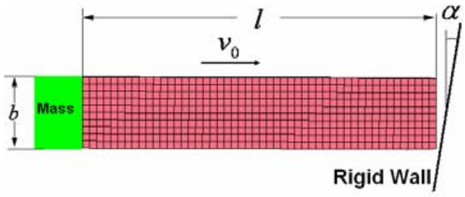


Figure 1 Model for numerical simulation

The finite element model used for numerical simulation is also shown in Fig. 1. The tube is modeled with a four-node shell element (Type 2 in LS-Dyna3D) using a piecewise linear elastic-plastic material model (Material 24 in LS-Dyna3D). The material is mild steel with the following mechanical properties: Young's modulus, E=205GPa; mass density, $\rho = 7.83E-6 Kg/mm^3$; initial yield stress, $\sigma_y = 220$ Mpa; and Poisson's ratio, $\nu = 0.3$. The engineering stress-strain hardening data are given in Table 1. The effect of strain rate is not considered in the current study.

Table 1 Stress-strain hardening data for mild steel

Plastic strain	Plastic stress (Gpa)
0.0000×10^{0}	0.220
5.9393×10^{-3}	0.250
1.6738×10^{-2}	0.300
2.2000×10^{-2}	0.332
2.5750×10^{-2}	0.353
5.1500×10^{-2}	0.370
1.0525×10^{-1}	0.374

EFFECTS OF UNCERTAINTIES IN CRASH PROCESS OF A STRUCTURE

Like other real-world engineering problems, vehicle crash is characterized by non-deterministic processes. Uncertainties and non-deterministic behavior are essential in these processes. Uncertainties can be grossly classified into three categories: parameter uncertainties, modeling uncertainties, and uncertainties in loading and boundary conditions. Parameter uncertainties include geometric properties uncertainties caused by manufacturing variances. Typical modeling uncertainties include mesh density uncertainty, material characteristic uncertainty, etc. Uncertainties in loading and boundary conditions are based on the fact that the crash event in different car accidents never is the same. Uncertainties in loading and boundary conditions are the most important and difficult aspect of the crash design since it may change the fundamental phenomena of a crash process; we will discuss this through examples in the following.

UNCERTAINTIES IN LOADING CONDITIONS

In a real vehicle crash event, the structure is rarely crashed along the pure axial direction of the structure. Experiments on the hat-type section column under oblique loads were conducted by Wallentowitz and Adam [11]. In [12], Han and Park found that there is a transition zone for the thin-walled tube to crash from the progressive collapse to the global bending, the value of the mean crush load drops to about 40% of the mean crush load in the pure axial collapse case after the angle is larger than the critical load angle. This means that uncertainties in loading condition can change the fundamental failure modes of the structure: from a high energy absorption failure mode (progressive collapse) to a low energy absorption mode (global bending collapse).

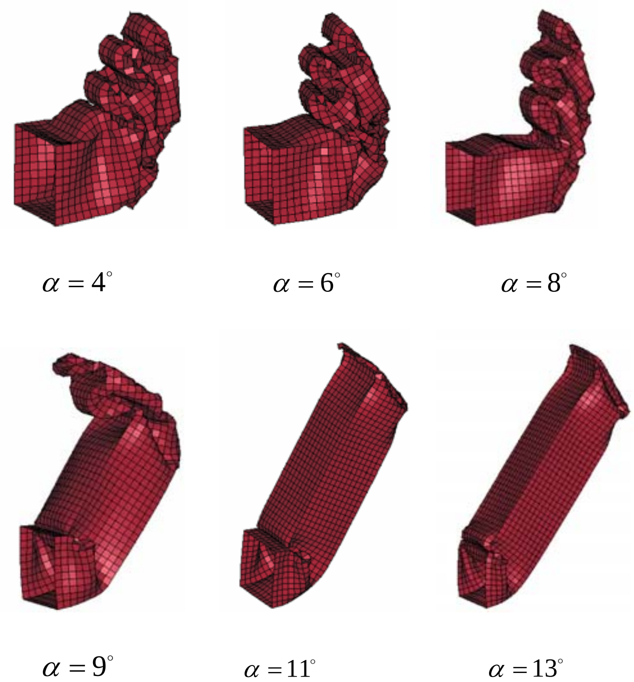


Figure 2 Deformed shapes with different crash angles

Figure 2 shows the deformed shapes of the thin-walled square tube under different crash angles. It is clearly seen a failure modes transformation from the progressive collapse to the global bending with the increase of the crash angle. Figure 3 depicts the time history of crash load for the different crash angles. The tube undergoes axial collapse mode until $\alpha = 8^\circ$. The global peak crash force takes place in the initial state of the deformation which is caused by the buckling of the tube, the crash force values then oscillate between local peak loads and local minimum loads due to the progressive collapse. At $\alpha = 9^\circ$, the crash force decreases after the third peak. For load angle greater than 9°, the crash force decreases continuously after the initial peak. The decrease in crash force is due to the global bending collapse shown in Fig. 2. A critical angle of $\alpha = 8^\circ$ is found to describe the failure modes transformation in this example.

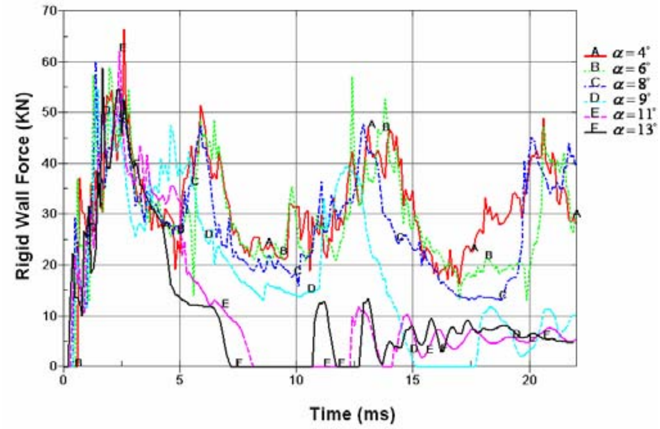
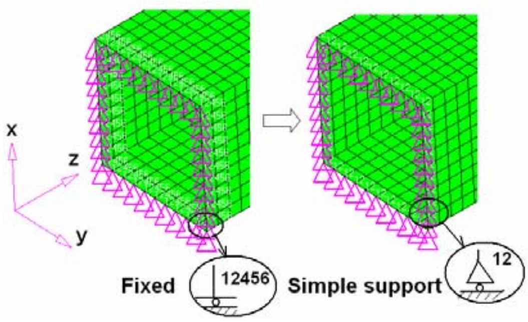


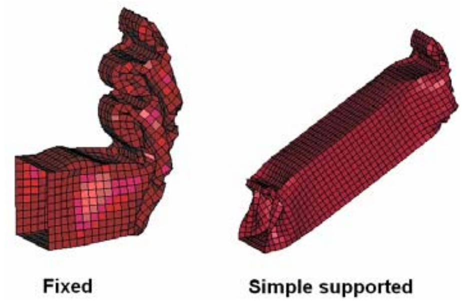
Figure 3 Crash force history with different crash angles

UNCERTAINTIES IN BOUNDARY CONDITIONS

Consider the boundary condition uncertainty which varies from a fixed boundary condition to a simple-supported boundary condition as shown in Fig. 4(a).



(a) Boundary condition uncertainty



(b) Deformed shape

Figure 4 Effects of boundary condition uncertainties

In the case of crash angle $\alpha = 8^\circ$, the boundary condition changes the failure mode of the tube from the progressive collapse to the global bending as shown in Fig. 4(b). This example elucidate that boundary condition uncertainty may change the fundamental phenomena of a crash process.

Consider another type of boundary condition uncertainty: uncertainties of the friction coefficient μ between the tube and the rigid wall during the crash process. Three values of μ are considered: $\mu = 0$ means frictionless between the tube and the rigid wall during the contact; $\mu = 1$ means that the tube is stick to the rigid wall once they contacted; $\mu = 0.5$ is a condition in between. The crash force histories for these three contact boundary conditions are shown in Fig. 5. It can be seen that boundary conditions with $\mu = 0.5$ and $\mu = 1$ induce the same magnitude of impact pulse at the very beginning of the crash. Frictionless sliding slightly reduces this impact pulse. This is because the tube end can deform freely along the rigid wall surface if no friction force exist. If we extended this boundary condition uncertainty into a real vehicle crash process, we may predict that a smooth surface of the energy absorption device is preferable because it can reduce the impact pulse endured by the passengers.

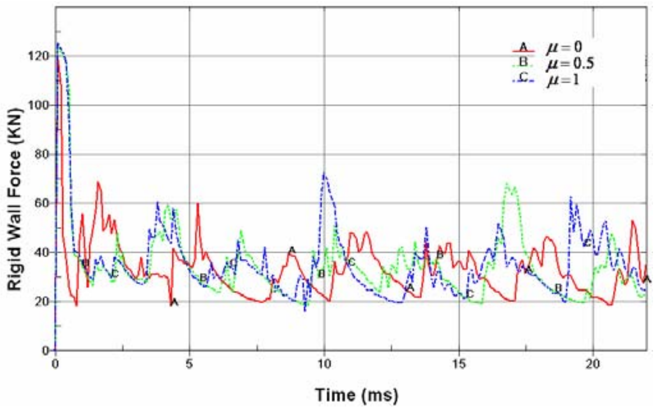
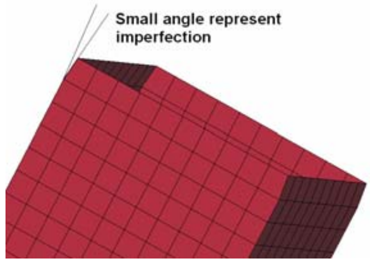


Figure 5 Crash force for different friction coefficients

UNCERTAINTIES DUE TO MODELING PROCESS

Modeling uncertainties came from information loss or inaccurate representation during the process of transforming a real engineering problem to a mathematical model. Finite element method is a numerical method in nature, numerical errors is another kind of uncertainty effect in the finite element modeling. For the crash analysis, bifurcation buckling is a typical phenomenon. It is difficult for the numerical method, such as the finite element method, to capture accurately the bifurcation point, the program may trace an impractical load curve based on an incorrect collapse mode as shown in Fig. 6 (a), in which all the side plates buckle outside, resulting a non-practical crash mode. To trace the correct path of post-buckling, imperfection (uncertainty) must be introduced to the numerical model to represent the buckling mode at the bifurcation point. A simple method is to introduce a very small angle at the end of the tube where crash load to be applied, as shown in Fig. 6(b). The normal mode of collapse is shown in Fig. 6(c). This example shows the great effects of modeling uncertainties on the crash analyses results.

(a) Incorrect collapse mode (without imperfection)

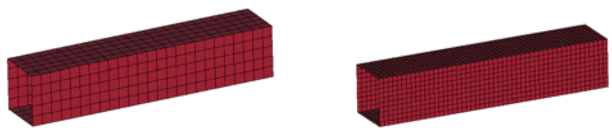


(b) Uncertainty (imperfection) introduced

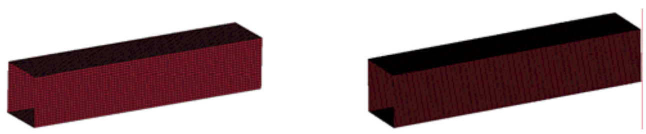
(c) Normal collapse mode (with imperfection)

Figure 6 Influence of modeling uncertainty

Another modeling uncertainty comes from the mesh density. Fig. 7 shows four levels of mesh density of the thin-wall square tube, higher level represents higher mesh density. The crash force histories are shown in Fig. 8, it can be seen that coarse mesh (mesh level 1) gives relative high global impact peak and the crash force. As the mesh density increases, the impact peak and the crash force converges to a unique result.



(a) Mesh level 1(20*25) (b) Mesh level 2 (40*50)



(c) Mesh level 3(80*100) (d) Mesh level 4(160*200)

Figure 7 Different mesh densities of thin-walled square tube

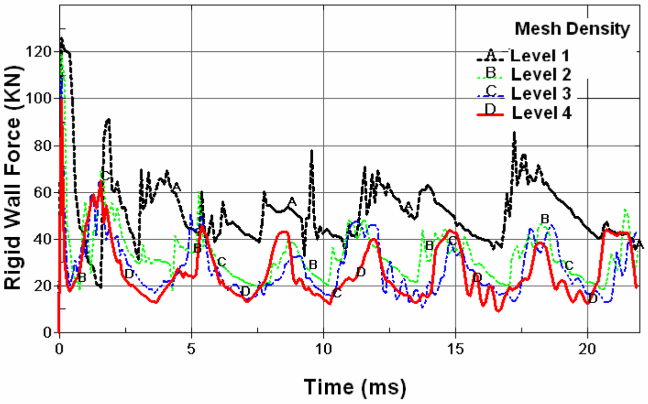


Figure 8 Crash force history for different mesh densities

OTHER UNCERTAINTY FACTS

Geometrical initial imperfection is a kind of parameter uncertainty which is discussed in reference [13]. Parameter uncertainties can be represented by the variation of structure dimensions due to manufacturing errors.

FUNDAMENTAL CRASH MECHANISMS

During a vehicle crash process, the behaviors of the energy absorption devices and other structures are very complicated. It involves nonlinear phenomena, including impact, large deformation, buckling and yielding, as well as nonlinear contact. For performing an effective design, it is necessary to investigate the crash mechanisms to gain an understanding of the process.

ELASTIC BUCKLING

Buckling failure is known as a major failure mode of thin-walled structures. Buckling modes have been used to design a structure for crashworthiness [14], but crashing is not equal to buckling. Structure may fail by yielding load far below the critical buckling load.

GLOBAL BENDING COLLAPSE THROUGH PLASTIC HINGE

Global bending collapse through plastic hinge is another major failure mode of thin-walled structure. Plastic hinge is a collapse mechanism which can be illustrated in Fig. 9. As shown in Fig. 9, as the load increase from P_1 to P_2, the fixed end of the beam yield first, the resultant plastic hinge results in that the beam becomes simply supported on both ends. As the load increases to P_{PCF}, which is the critical collapse force, another plastic hinge generated at the location where the force is applied and the structure failed eventually.

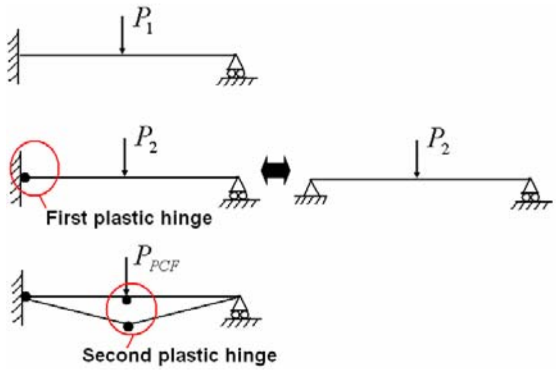
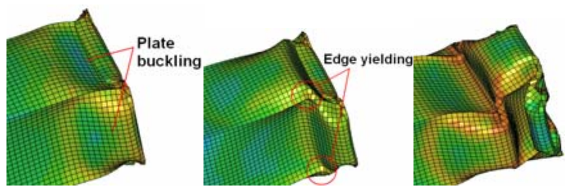


Figure 9 Structure fails through plastic hinge

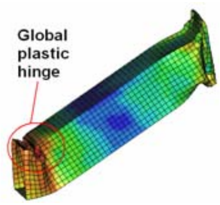
FAILURE MECHANISM OF THIN-WALLED SQUARE TUBE

The square tube fails by the combination of elastic buckling and plastic hinge as shown in Fig. 10. At the beginning of the impact, the crash force increase rapidly, and local buckling occur at the weakest plates of the tube (Fig. 10(a) (i)). As the deformation of the plates increase, stress concentrates at the local edges results in edge yielding (Fig.10 (a) (ii)). The tube then collapses and its plates are folded according to the preceding buckling shape (Fig.10 (a) (iii)). The second buckling and yielding then occur. The successive load peaks are lower than the first one because the deformed part from the first local buckling works as trigger in the following deformation. When oblique load applied with an angle greater than the critical angle, instead of local edge yielding to form local plastic hinge, the plate at the maximum moment yield to form a global plastic hinge (Fig. 10(b)), the tube then rotate around the global plastic hinge which provides little energy absorption.



i)Local buckling ii) Local plastic hinge iii) Progressive collapse

(a) Axial progressive collapse



(b) Global bending collapse through plastic hinge

Figure 10 Failure mechanism of thin-walled square tube

DESIGN FOR CRASHWORTHINESS WITH LOAD DIRECTION UNCERTAINTIES

Since the progressive collapse is much more preferable for the energy absorption purpose, the problem then becomes how to trigger progressive collapse mode in a relatively large crash angle, i.e., how to achieve a robust design against load direction uncertainty. To prevent a global bending failure mode, one obvious design idea is to add more materials to the location where global plastic hinge tend to appear. The dynamic crash analyses as above can be used to determine the location where the material should go. One drawback with this is the high computational cost and the complicated design task.

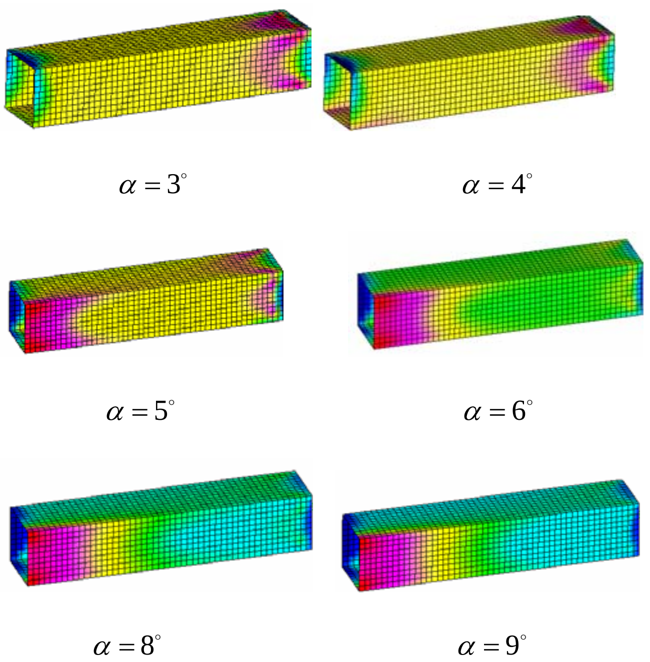


Figure 11 Stress distribution transformation process

Instead, static analyses can also be carried out to investigate the critical condition for the collapse mode. Figure 11 shows the major principal stress distribution for different load angles. It is seen that there is a stress distribution transition process from the front end of the tube to the fixed end with the increase of the load angle. This corresponds to the transition from progressive collapse mode to global bending mode. By the static analyses results, we can predict that the global plastic hinge tend to appear at the location where stress is relatively high. To prevent this global plastic hinge, more material needs to be added to the region where stress is high. One simple method is to increase the wall thickness of the tube at this portion. Without increase the total amount of material, the wall thickness at the locations where stresses are relatively small can be reduced. The design philosophy is shown in Fig. 12.

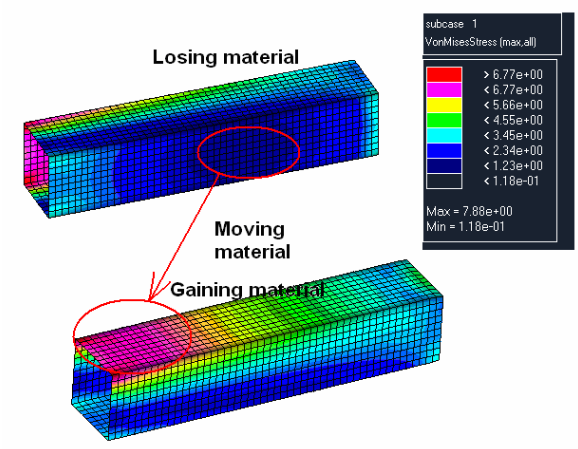


Figure 12 Design philosophy

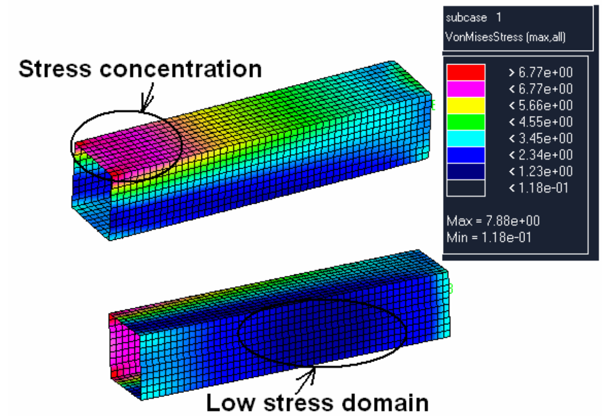


Figure 13 Stress distributions with original design

By using this idea, the wall thickness at the fixed end of the tube is increased from the original $t = 1.5mm$ to $t_1 = 2.0mm$ to reduce the stress level. Accordingly, the wall thickness is decreased to $t_2 = 1.2mm$ at the location where stress is relatively small. This layout change is based on the static analysis result of the original tube under an oblique load at $\alpha = 9^\circ$, as shown in Fig.13. This design is called Design 1 as shown in Fig. 14. A static analysis of Design 1 is then carried out and the stress distribution is shown in Fig. 15. It can be seen that the stress concentration location has transferred to new location at the edge of the increased wall thickness domain. This implies a new potential location of plastic hinge in the dynamic crash process. Same idea is applied to increase the wall thickness at the new stress concentration location by define new domain with increased wall thickness $t_3 = 1.7mm$. Another domain with the decreased wall thickness $t_4 = 1.1mm$ is defined at the location where stress level is relatively low. The total mass still remains unchanged during this process. This new design is represented as Design 2, which is shown in Fig. 16. Figure 17 depicts the stress distribution of Design 2 by static analysis, it can be seen that the stress distribution is improved and the stress concentration effect is relieved. This process can be repeated until a satisfied design is obtained as illustrated in Fig.18.

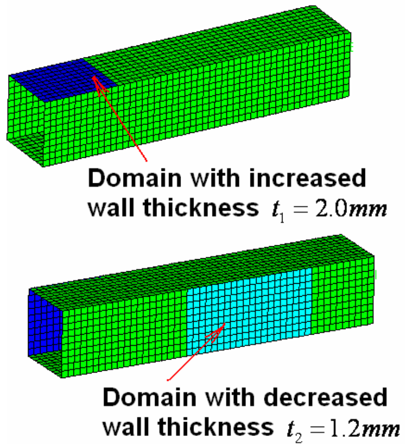


Figure 14 Design 1

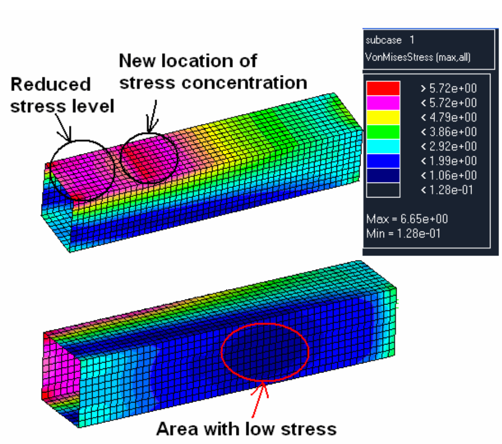


Figure 15 Stress distributions with Design 1

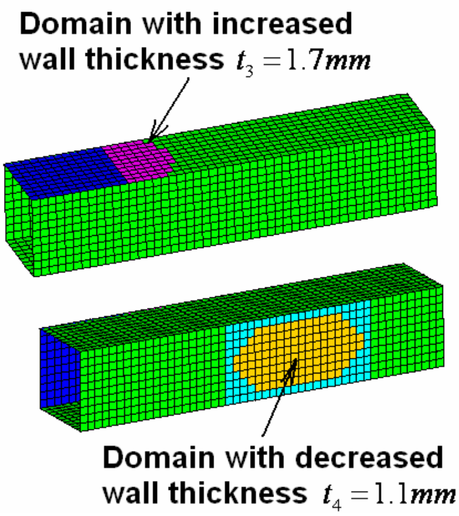


Figure 16 Design 2

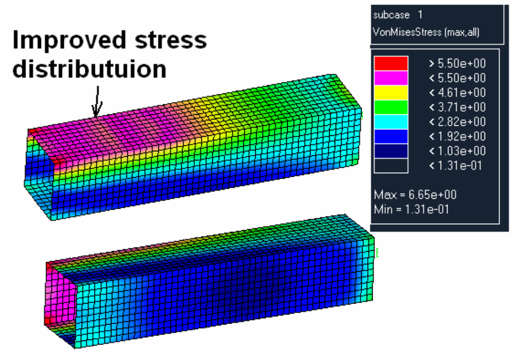


Figure 17 Stress distributions with Design 2

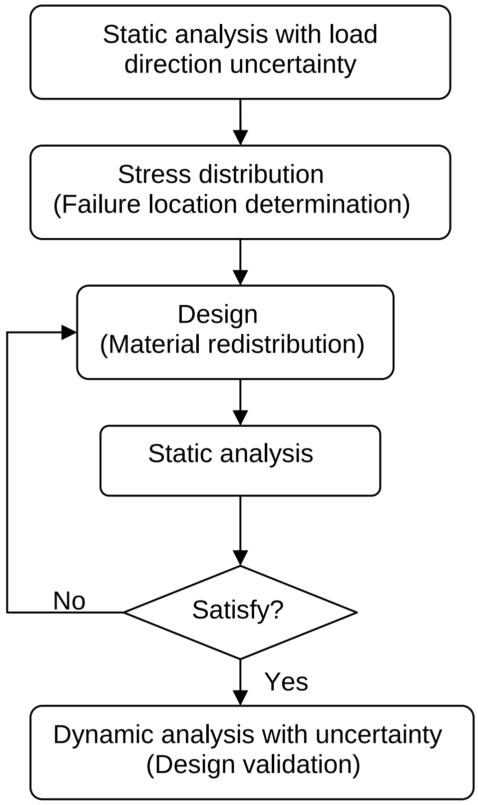
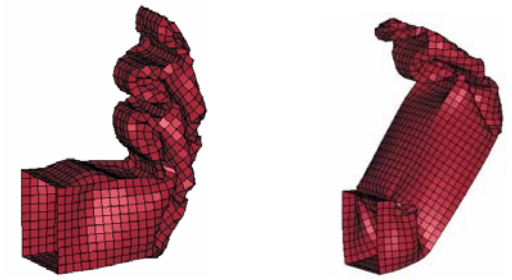


Figure 18 Design procedure

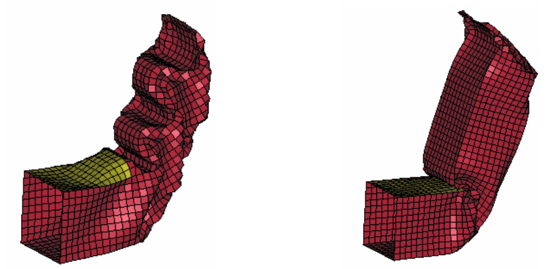
Figure 19 shows the transition point from the progressive collapse to the global bending for Design 1, Design 2 and the original design. Design 1 increases the critical angle to 12°, while Design 2 further increases this to 13°. The crash force of Design1, 2 and the original design for load directions $\alpha = 9^\circ$ and $\alpha = 13^\circ$ are compared in Fig. 20. For load direction $\alpha = 9^\circ$, Design 1 shows one more oscillation of the crash force resulting in more energy absorption. But for the increased load angle $\alpha = 13^\circ$, both the original design and Design 1 are characterized by the direct decrease of the crash force after the first peak, both are caused by the global bending mode shown in Fig. 19. Design 2 instead shows continuous oscillation of load for both load angles which implies more energy absorption capability.

From the above process, it can be seen that a step by step failure mode modification can be carried out by using the simplified static analyses which results in less computational cost. This approach can be extended to more general structural design process including the use of topology optimization. Note that the purpose of this example problem is to demonstrate the general approach for an effective and efficient topology optimization.



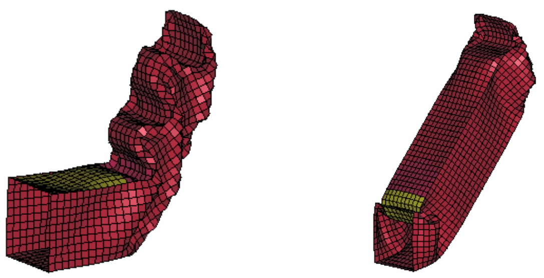
$\alpha = 8^{\circ}$ $\alpha = 9^{\circ}$

a) Original design



$\alpha = 12^{\circ}$ $\alpha = 13^{\circ}$

b) Design 1



$\alpha = 13^{\circ}$ $\alpha = 14^{\circ}$

c) Design 2

Figure 19 Critical angles for different designs

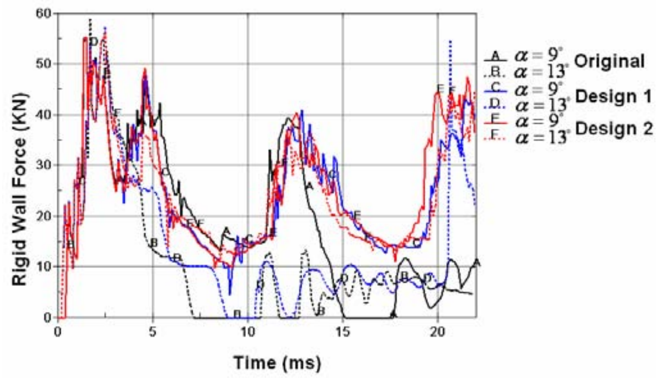


Figure 20 Comparisons of rigid wall forces for different Designs

CONCLUSION

The following conclusions are obtained:

1) Uncertainties have great effects on a crashworthiness design. Uncertainties in loading and boundary conditions can change the fundamental crash phenomena. Uncertainties in modeling also show great effects on the simulation results.

2) An energy absorption device such as the square thin-walled tube fails by two major crash modes: progressive collapse by local buckling and edge yielding and global bending collapse by forming global plastic hinge. There exists a critical loading angle for the structure to crash with the progressive collapses, which results in more energy absorption.

3) A step by step failure mode modification approach using static analyses can be effective and efficient for the crashworthiness design problem. Our future research is to extend this approach into topology optimization for more general crashworthiness design problems.

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